MEASURING RELIABILITY

(03)

INTRODUCTION

- Reliability is the probability of successful performance of a system at any time.
- The successful performance of any system depends on the extent to which reliability is designed and built into it.

Why Beneficial - Problem:

- Seemingly identical systems, operating under similar conditions, fail at different times.
- Failure of such systems - probabilistic.
- Most engineering systems are composed of multiple components; the system reliability must be carefully analyzed and understood to have a useful system.
- Various measurement techniques and data reduction techniques will greatly improve our ability to analyze data.

OBJECTIVES:

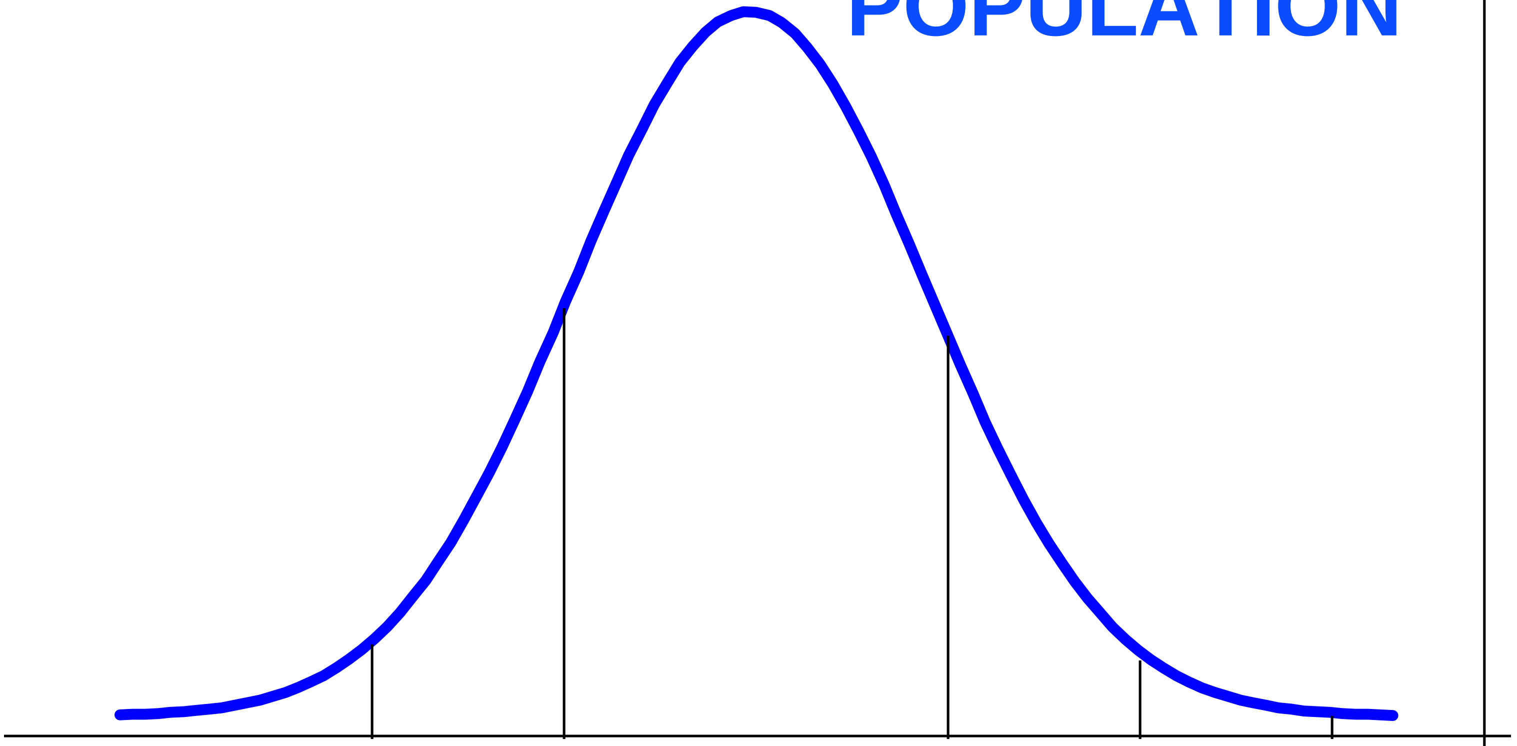
- **Be able to answer (or know):**
- **What is the difference between a population and a sample? What are the requirements for taking a sample?**
- **What are the general procedures for analyzing data?**
- **What is the difference between the sample standard deviation and the population standard deviation?**
- **What are the differences between discrete and continuous variables (see also next section)?**
- **What is the normal distribution, the standard normal distribution?**
- **Be able to analyze data using the normal distribution.**

OUTLINE

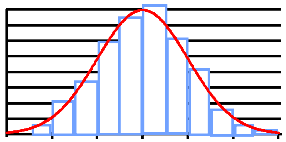
- Definitions.
- Predicting Reliability (general methodology).
- Measures of Central Tendency.
- Measures of Dispersion.
- Normal Distribution.
- Histograms.
- Solving Problems with the Normal Distribution.
- Additional Information.

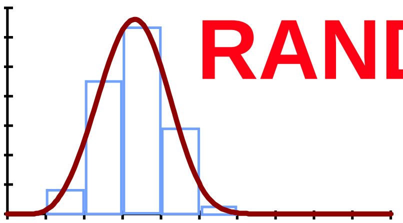
DEFINITIONS:

POPULATION



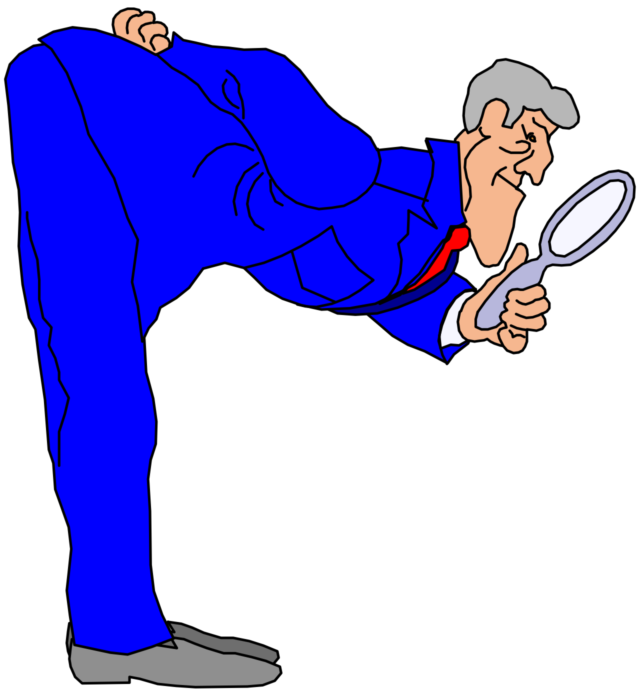
SAMPLE





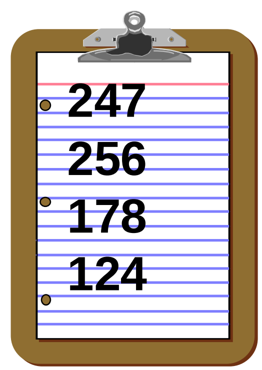
RANDOM SAMPLE?

DEFINITIONS (con't)



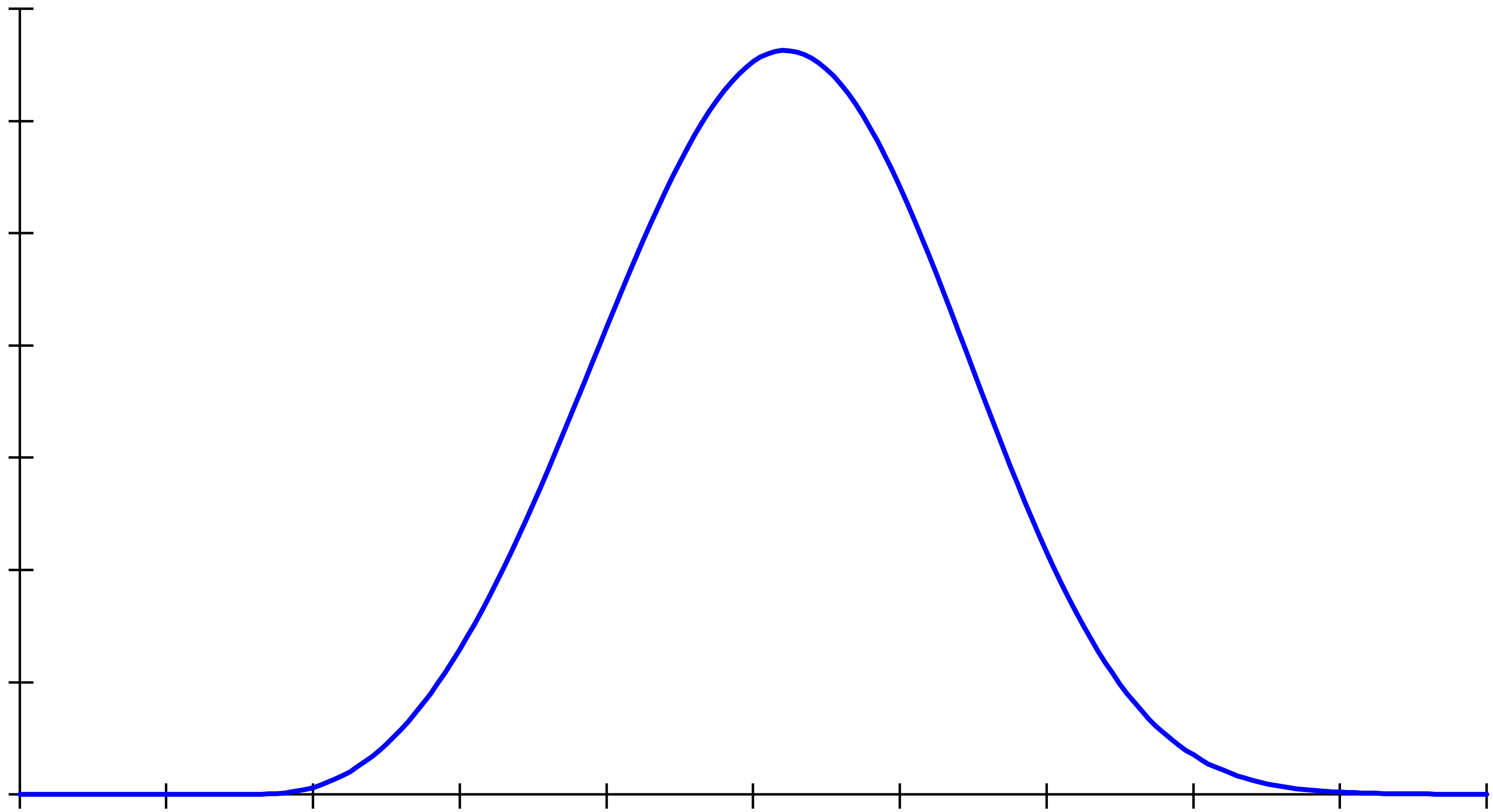
EXPERIMENT, X

EVENT, x



DEFINITIONS (con't)

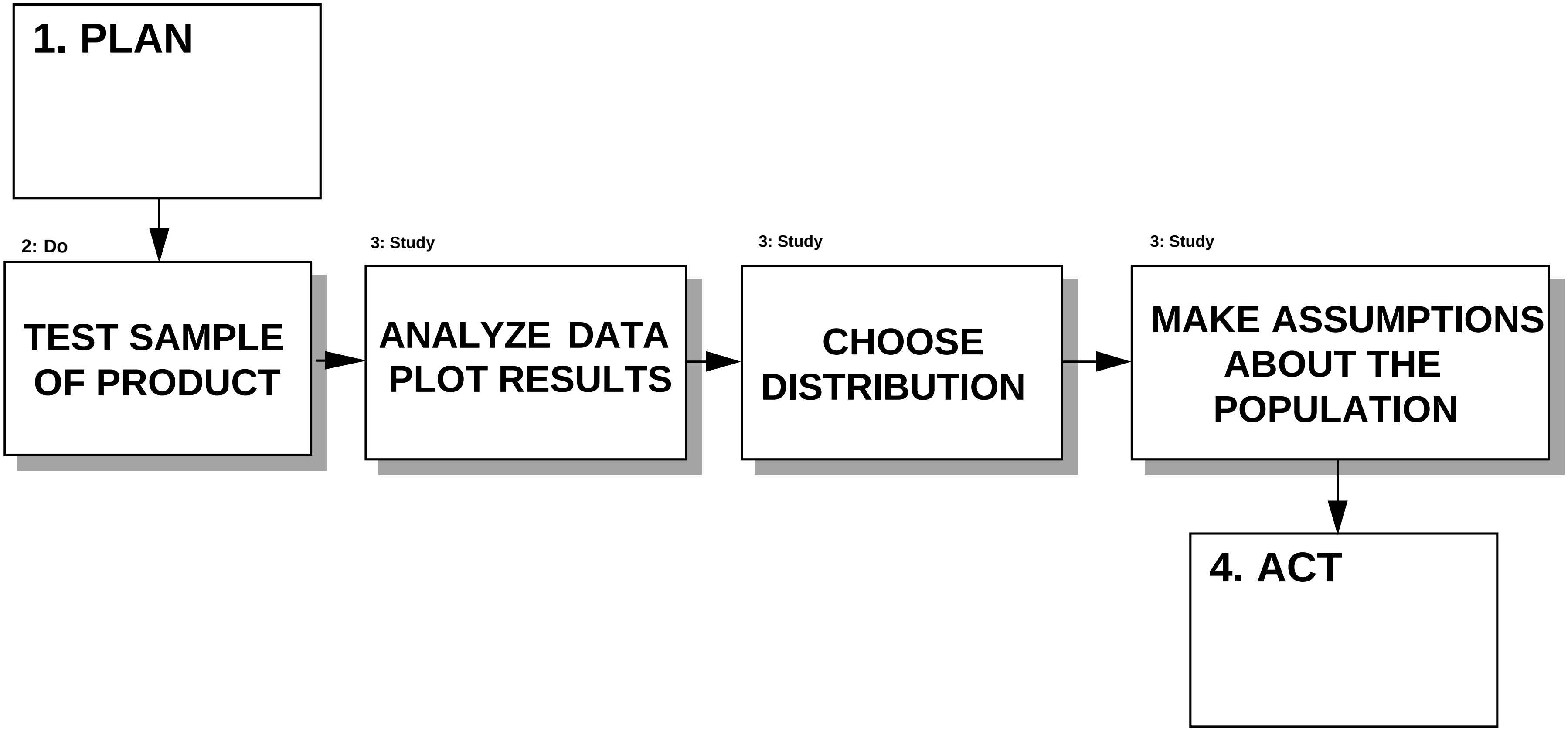
DISTRIBUTION FUNCTION



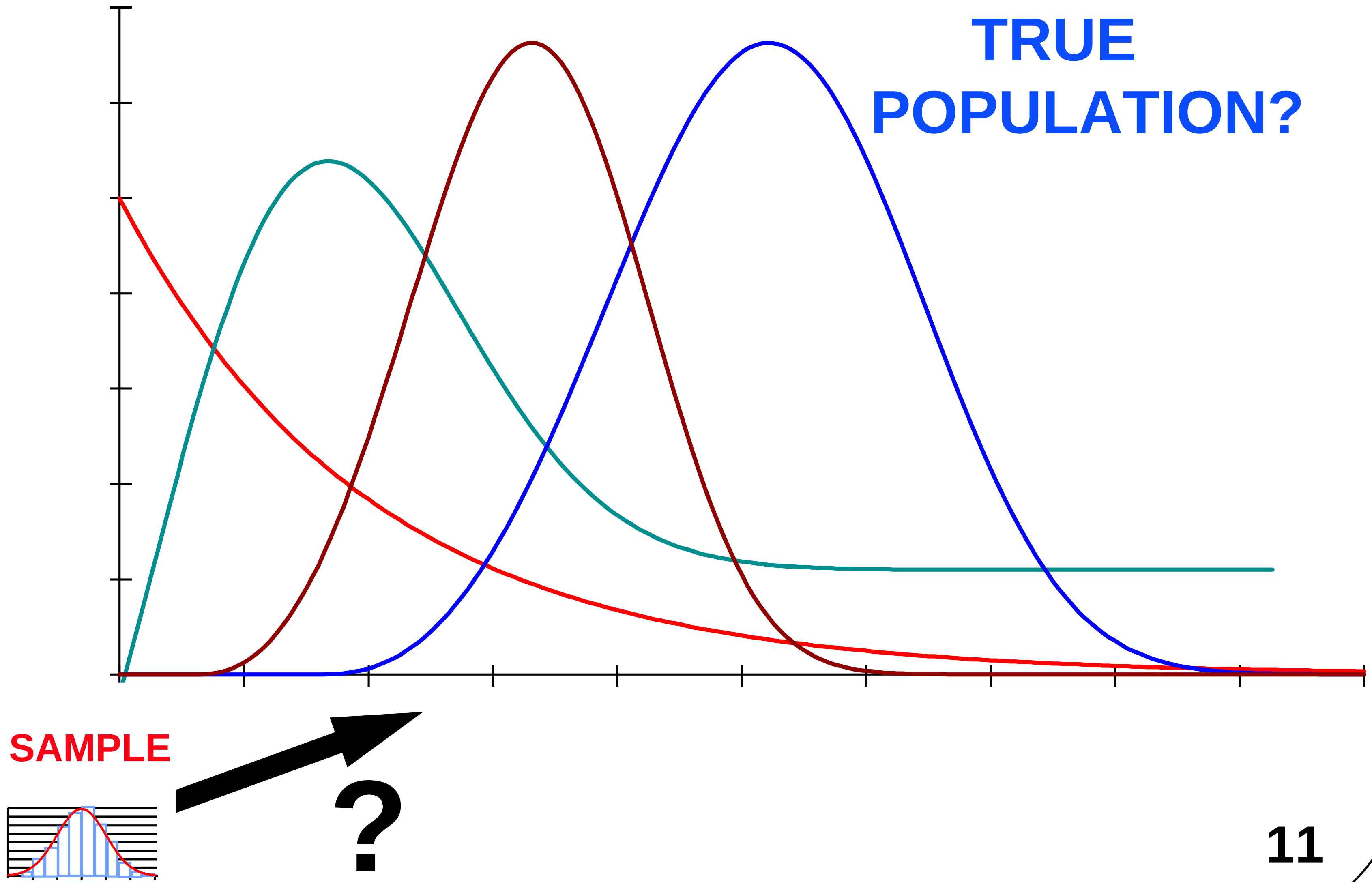
TO PREDICT RELIABILITY WE NEED:

- **TEST A SAMPLE OF THE PRODUCT -- we need to get TEST DATA**
- **WE NEED TO ANALYZE THE DATA.**
- **WE THEN ATTEMPT TO DESCRIBE THIS TEST DATA MATHEMATICALLY--OFTEN BY FITTING THE DATA TO A DISTRIBUTION FUNCTION THAT DESCRIBES IT.**
- **WE CAN THEN MAKE ASSUMPTIONS ABOUT THE POPULATION FROM WHICH THE SAMPLE WAS TAKEN.**

TO PREDICT RELIABILITY WE NEED TO:



WHAT TYPE OF DISTRIBUTION DOES THE TRUE POPULATION FIT?



TYPES OF DISTRIBUTIONS:

- NORMAL
- LOGNORMAL
- EXPONENTIAL
- GAMMA
- WEIBULL
- EXTREME VALUE
- CAUCHY

POISSON
BINOMIAL
NEGATIVE BINOMIAL
HYPER GEOMETRIC
RALEIGH
BETA

THIS IS SIMILAR TO ANY TYPE OF STATISTICAL SAMPLING.

- **A sample opinion poll is similar to testing sample of products.**
- **A sample is taken and the average of the sample is said to represent the average of the population from which the sample was taken (within tolerance limits).**

ANOTHER STATISTICAL SAMPLE IS AS FOLLOWS

- **A sample opinion poll is similar to testing a sample of a product.**
- **A sample is taken and the average of the sample is said to represent the average of the population from which the sample was taken (within tolerance limits).**
- **A histogram is plotted of the peoples responses.**
- **This histogram (the sample or test data) is fitted to a frequency distribution function (in this case a Normal Distribution).**

A STATISTICAL SAMPLE OF PRODUCT IS SIMILAR:

- A retail shoe chain wants to know what sizes of shoes to stock.
- A sample is taken and the average of the sample is said to represent the average of the population from which the sample was taken (within tolerance limits).
- A histogram is plotted of the shoe sizes (we ignore widths).
- This histogram (the sample or test data) is fitted to a frequency distribution function (in this case a Normal Distribution).

NORMAL DISTRIBUTION

BASIC STATISTICAL TERMS-- Measures of Central Tendency

- **MODE, the most common item or event**
- **MEDIAN, the middle number or position in a series of ordered items or events.**
- **MEAN, Arithmetic, X_{bar}, An average of a series of quantities or values:**
- **X_{bar} =**

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

BASIC STATISTICAL TERMS-- Measures of Dispersion

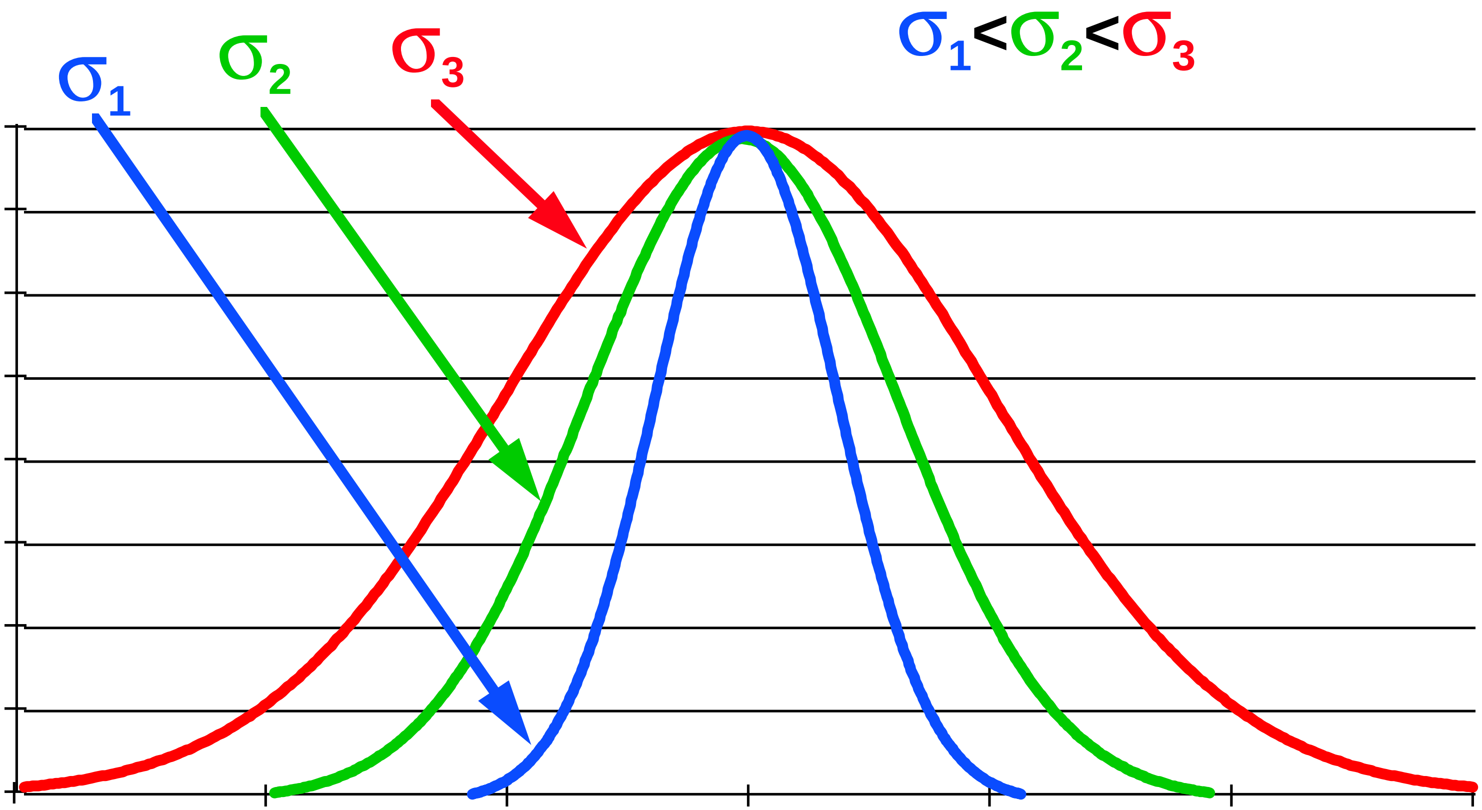
- **STANDARD DEVIATION is a measure of the dispersion or scatter of values of the random variable about the mean ($\overline{X}$, X_{bar} or μ).**
- **POPULATION STANDARD DEVIATION**

$$\sigma = \{\Sigma(X-\mu)^2/N\}^{1/2}$$

SAMPLE STANDARD DEVIATION

$$s = \{\Sigma(X-\overline{X})^2/(n-1)\}^{1/2}$$

STANDARD DEVIATION



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PROBLEM: Statistical Review

- Find the mean, mode, median & standard deviation of: x = 17, 45, 38, 27, 6, 38, 11, 54 and 34
- x - avg. = x-avg. (x-avg)^2
- _____ - _____ = ______ _______
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_____ = Σ(x) _______ = Σ[(x-avg)^2]

(P03-1)

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PROBLEM: Statistical Review

- **Find the mean, mode, median & standard deviation of: x = 17, 17, 19, 21, 37, 23, 26, 26, 21 and 33**
- **x - avg. = x-avg. (x-avg)^2**
- _____ - _____ = _____ _____
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- _____ - _____ = _____ _____

_____ = Σ(x) _____ = Σ[(x-avg)^2]

A STATISTICAL SAMPLE OF PRODUCT IS SIMILAR:

- **A valve manufacturer wants to know the average life of a certain model valve (A14).**
- **A sample is taken and the average of the sample is said to represent the average of the population from which the sample was taken (within tolerance limits).**
- **A histogram is plotted of the peoples responses.**
- **This histogram (the sample or test data) is fitted to a frequency distribution function (in this case a Normal Distribution).**

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PROBLEM --ANALYZE DATA

- **DEMONSTRATION & HANDOUT**
- **A Manufacturer wants to know the average life of a new model valve (A14).**
- **From the population test a sample of the lives of 30 valves.**
- **Analyze the data, find the average, order the data and plot a histogram.**
- **Draw a continuous line through the histogram.**
- **[Determine the distribution that best fits the data.]** **(Assumed here to be normal distribution)**
- **Make assumptions about the population.**

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PROBLEM -- DATA VALVE FAILURE:
Cycles to first failure - A14 valve
RANK DATA, FIND AVG. & S.D

7500	6840	14630
11660	7190	9860
13070	8560	11990
8110	9240	15820
11000	12630	12030
9450	9560	16030
12430	13140	10490
10490	14950	10310
11540	10510	11270
13270	10510	12490

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PROBLEM -- DATA VALVE FAILURE:
Cycles to first failure
RANK DATA, FIND AVG. & S.D

6840	10490	12430
7190	10490	12490
7500	10510	12630
8110	10510	13070
8560	11000	13140
9240	11270	13270
9450	11540	14630
9560	11660	14950
9860	11990	15820
10310	12030	16030

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PROBLEM -- DATA VALVE FAILURE:
Find Count of Data in Each Interval

Minimum	Maximum	Rank (i)	Count
4000	4999	1	______
5000	5999	2	______
6000	6999	3	______
7000	7999	4	______
8000	8999	5	______
9000	9999	6	______
10000	10999	7	______
11000	11999	8	______
12000	12999	9	______
13000	13999	10	______
14000	14999	11	______
15000	15999	12	______
16000	16999	13	______
17000	17999	14	______

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PROBLEM -- DATA VALVE FAILURE: FIND AVG. & S.D

Minimum = ______

Maximum = ______

Sum = 336570

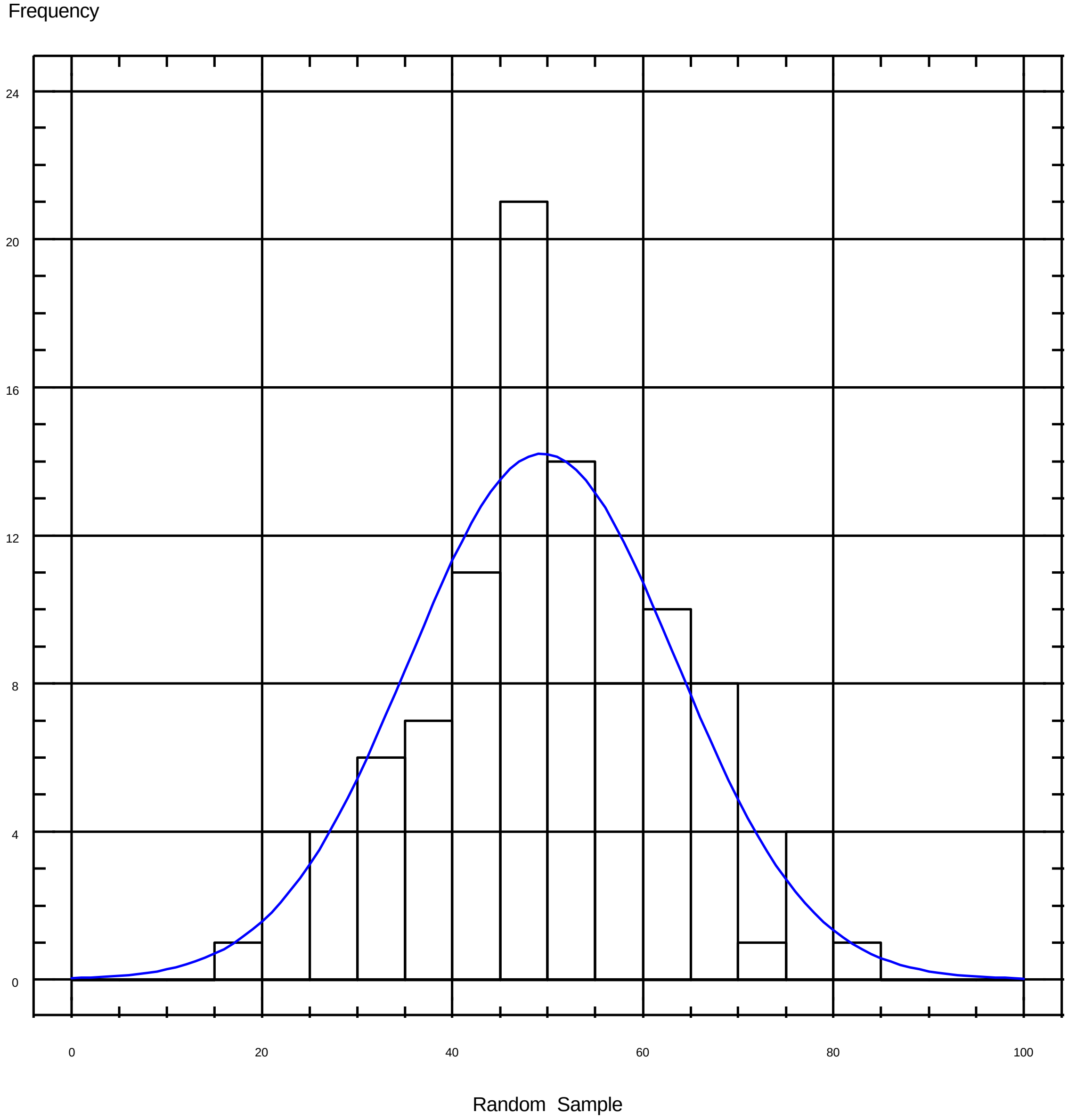
Count = ______

Average = ______

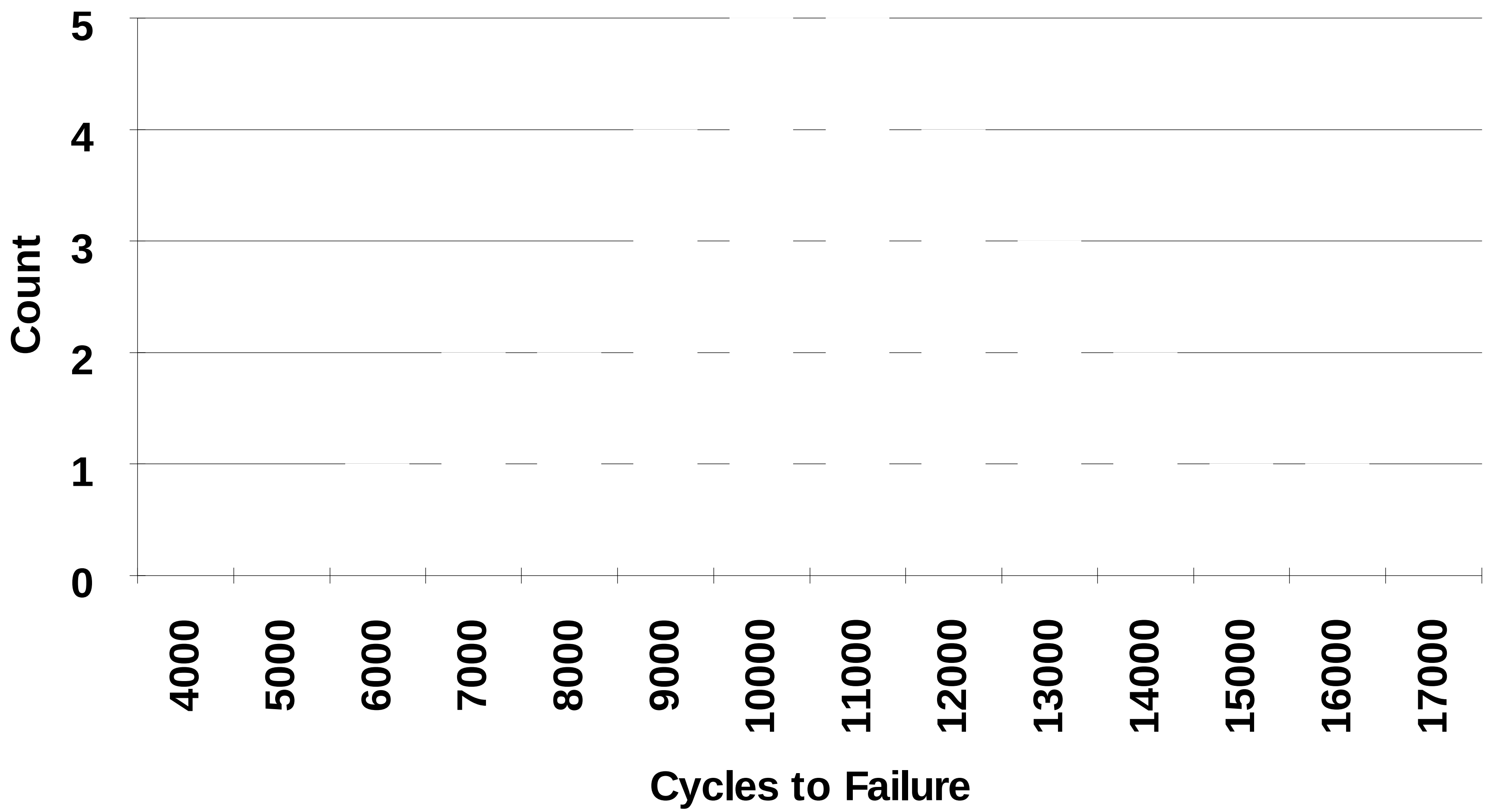
Std.Dev = 2369.1

(Is this the population or sample standard deviation)

FREQUENCY DISTRIBUTION (Histogram)



Problem: FREQUENCY DISTRIBUTION (Histogram)



NOW THAT WE'VE GOTTEN THIS FAR WE WANT TO:

- **Go into more detail on the Normal Distribution Function.**
- **We know the average of the population.**
- **Now we want to know other things:**
  - **We want to know the probability that the population (of shoe sizes, valves, etc.). is within certain limits.**
    - » **what percent of the valves fail in less than 8000 cycles.**
    - » **what percent of shoe sizes are greater than 12s**

PROBLEM SOLVING OPTIONS

- We could empirically develop an equation for our plot and integrate under the area in question.
- We could manually calculate the area under the plot.
- We could choose a distribution which we think represents the data.

Normal Distribution

Normal Distribution Function:

The equation for the function is as follows:

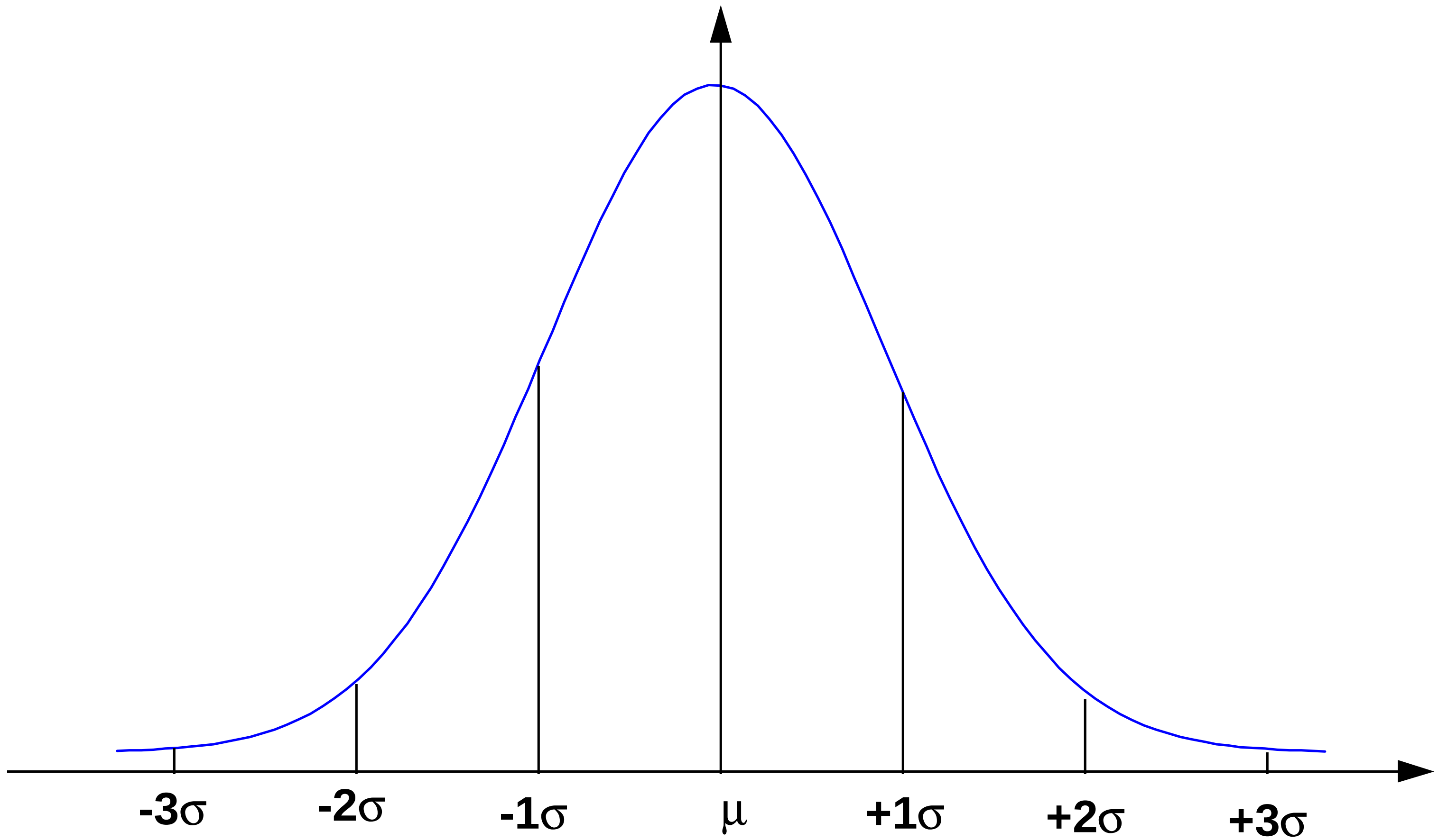
$$Y = f(x) = [\ 1 / \sigma\ (2\pi)^{1/2}]\ \ exp\ (-1/2)[(x-\mu) / \sigma]^2$$

Problem with Names: NORMAL DISTRIBUTION, STANDARD NORMAL DISTRIBUTION (special case), NORMAL CURVE, GAUSSIAN DISTRIBUTION all mean the same thing.

Normal Distribution (con't)

- The Normal Distribution is a continuous distribution from - INFINITY to + INFINITY and symmetrical about the mean. The total area under the curve is equal to one. The point of inflection on the curve is equal to one standard deviation from the mean .
- The area under the curve for ± 1 is = 0.683.
- The area under the curve for ± 2 is = 0.9545.
- The area under the curve for ± 3 is = 0.9973.

Normal Distribution



PROBLEM SOLVING STRATEGIES

- **The equation for the normal distribution function is difficult to work with.**
- **If we normalize it by having developing another equation where the mean is 0 and the standard deviation is 1, we can develop tables to show areas instead of calculating them every time.**

Standard Normal Distribution

Convert to the standard normal distribution:

The Standard Normal Distribution is a normal distribution where $\mu = 0$ and $\sigma = 1$.

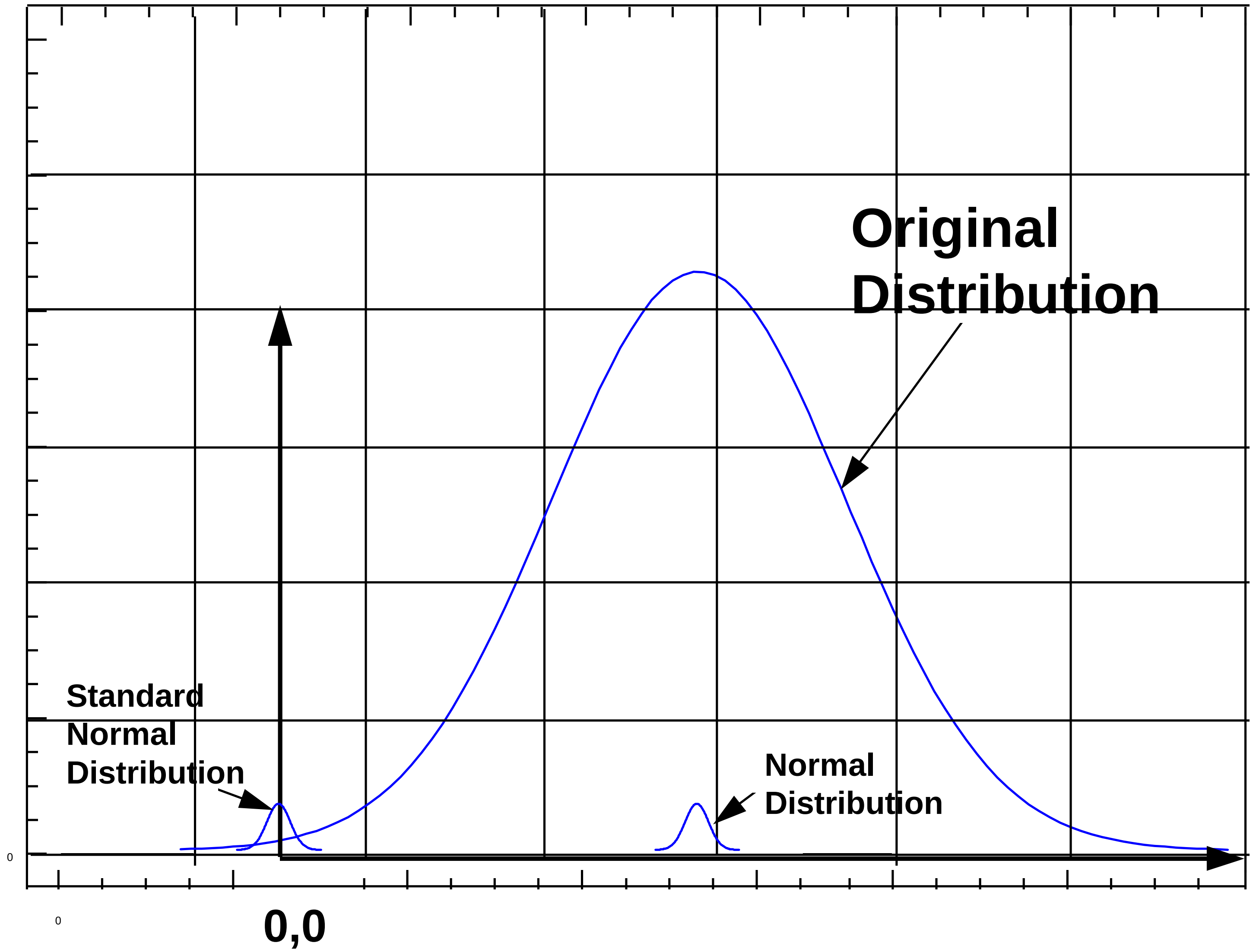
The number of standard deviations (z) for an equivalent Standard Normal Distribution is determined by the following :

$$z = (X - \mu)/\sigma$$

The value z is used to determine the area under the Standard Normal Distribution and the probability of occurrence for a particular range of t [or x]. The equation is as follows:

$$Y = f(x) = [\,1 / (2\pi)^{1/2}]\ \ exp\,(-1/2)(\,z\,)^2$$

RELATIONSHIP OF DISTRIBUTIONS



Standard Normal Distribution
(con't)

- To use the normal distribution tables to further analyze the population we have to decided:
  - Is this a "one-tailed" test: do we want to find out if the population is ONLY LESS THAN SOME VALUE (e.g. what percent of valves fail in less than 6,000 cycles.)
  - Is this a "two-tailed" test: do we want to find out if the population falls WITHIN A RANGE. (e.g. a time delay relay must actuate in at least 90 seconds but no more than 98 seconds).

Standard Normal Distribution
(con't)

- HINTS:
- DRAW CURVE OF A NORMAL DISTRIBUTION WITH μ AND σ.
- HIGHLIGHT PORTION OF CURVE YOU WANT TO FIND.
- LABEL THE CURVE "PASS," "FAIL," etc.
- SOLVE FOR HALF OF THE CURVE AT A TIME.
- REMEMBER ONE-HALF OF THE AREA = 0.5
- SUM RESULTS, CHECK FOR REASONABLENESS.

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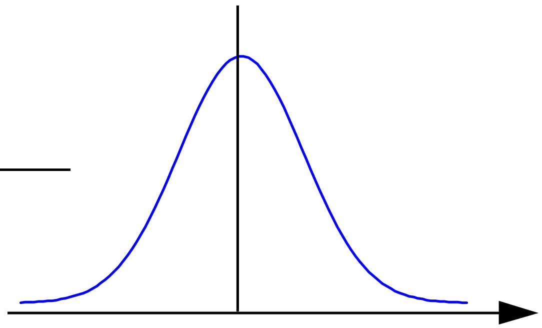
One Limit Problem

- Exploding bridgewire power supply must produce at least 1500 V. (EX. 9 p. 60).

  μ = 1575 and σ = 46.
- Lower limit:
- z = (______-______)/(______) = ________
- From one tail table on page 61
- Lower tail area = _______



- The probability that the range is greater than 1500 V is = _______
- The probability that the range is less than 1500 V is = 1 - _______ = _______

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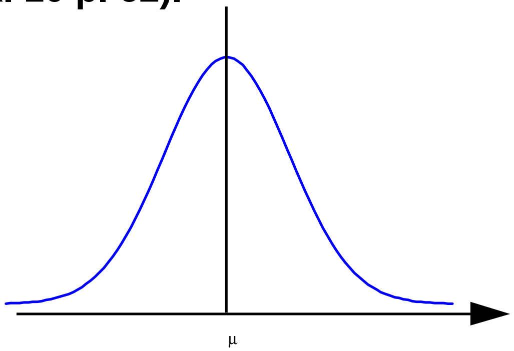
Non-Symmetrical Two Limit Problems

- A time delay relay is required to delay at least 90 sec. but no more than 98 sec (EX. 10 p. 62).

  μ = 95 and σ = 2.2



- Lower limit:
- z = (_____-_____)/(_____) = _____
- From one tail table on page 63-64
- Lower limit tail area = _____
- Upper limit
- z = (_____-_____)/(_____) = _____
- From one tail table on page 63-64
- Upper limit tail area = _____
- The probability for the range from 90 sec. to 98 sec. is therefore:
- = ________

WE ALSO WANT TO LOOK AT (in a following section):

- **From the failure distributions (or probability density functions) we “derive” the following:**
- **Reliability Function--the probability that an item will survive for a stated interval.**
- **Hazard function-- the instantaneous failure rate of an item.**

CONCLUSIONS -- MEASURING RELIABILITY

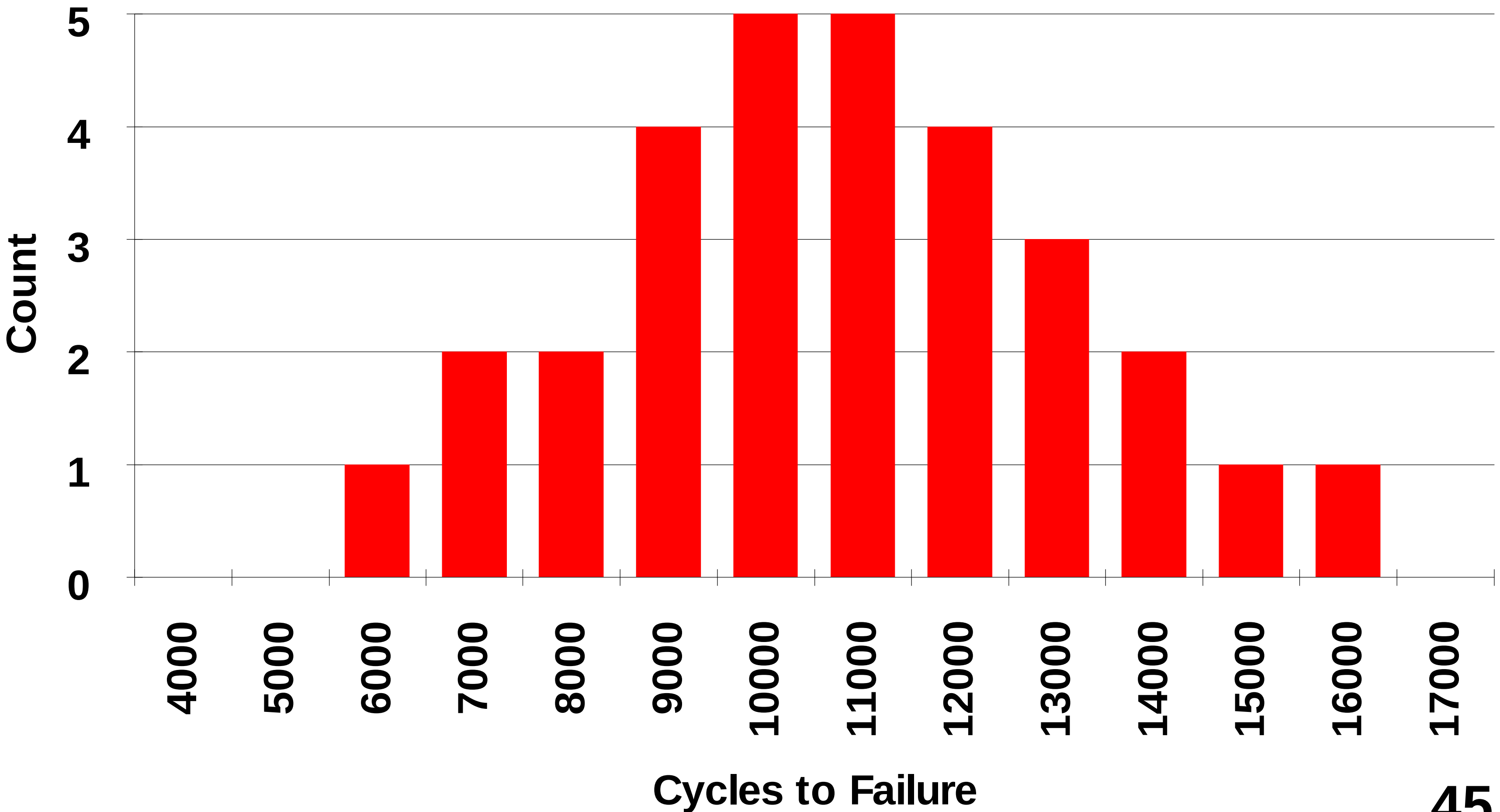
- TAKE SAMPLE OF RELIABILITY DATA.
- ANALYZE THE DATA.
- MAKE ASSUMPTIONS ABOUT THE POPULATION.
- CHOOSE A DISTRIBUTION THAT FITS THE DATA.
- THESE MEASUREMENTS WERE PERFORMED TO GAIN KNOWLEDGE ABOUT THE PRODUCT AND ITS RELIABILITY PERFORMANCE.

END>

ADDITIONAL INFORMATION

<C

Problem: FREQUENCY DISTRIBUTION (Histogram) A14 VALVE



PROBLEM: --ANALYZE DATA

- DEMONSTRATION & HANDOUT
- A Manufacturer wants to know the average life of a new model of solenoid valve.
- From the population test a sample of the lives of 100 solenoid valves.
- Analyze the data, find the average, plot a histogram.
- Draw a continuous line through the histogram.
- Determine the frequency distribution that best fits the data.
- Make assumptions about the population.

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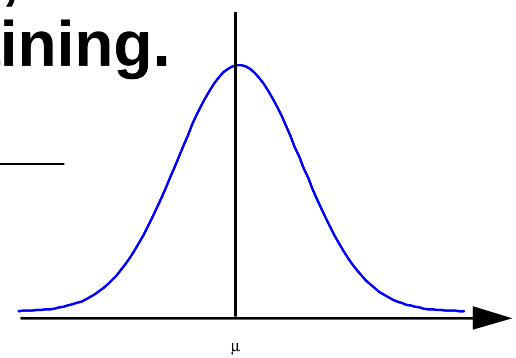
PROBLEM: ANALYZE VALVE

- A Manufacturer wants to know the average life of a new model of solenoid valve.
- He/She also wants to know what is the probability of or percent of valves that will fail in less than 6000 cycles.
- We have found the average valve life, μ, to be 11,110 cycles and the standard deviation, σ, to be: 2588.4 (cycles).
- $Z = (X-\mu)/\sigma$ = (_____ - _____) / _____ = _____
- this is a “one-tailed test”, use table 5-6 on page 63 of Reliability Training.
- The probability = ________



DEFINITIONS

POPLULATION

The entire group of individuals or objects.

ALL of a particular type of product (All Model 12F Valves)

ALL of a particular group of people (The total US population)

SAMPLE

Some "sample" of the population.

Some subset of measurements or observations from a larger set of values, the population.

RANDOM SAMPLE (of a population).

There is no bias (or favoritism) in taking the sample

If $x_1, x_2, ..., x_n$ are independent and identically distributed random variables.

DEFINITIONS (continued)

EXPERIMENT

The measuring of some "quantity" (X).

E.g. Measuring the weight of each person getting on an airplane.

EVENT

A particular measurement value taken in an experiment. (x)

E.g., the weight of the last person getting on the plane is 247 pounds.

The possible outcome of an experiment or measurement of a quantity X (that take on varies values of x), within the range of - infinity to + infinity.

X or other random variable uses capital letters.

x (uses lower case letters) is a particular value taken of X.

DEFINITIONS (continued)

DISTRIBUTION FUNCTION

"A mathematical formula that describes a curve (of a distribution)."

A function of a variate for which the functional value corresponding to a given value of the variate is the number of or the proportion of values of the variate equal to or less than the value.

VARIATE

A variable quantity associated with a probability distribution or a random variable.

The function given by:

$$F(x) = P(X <= x) = Integral\ f(t)dt$$

from - infinity to + infinity and where f(t) is the value of the probability density function of X at t.

and if X is a continuous random variable.

DEFINITIONS (CONTINUED)

The Normal Distribution is a function that describes the distribution of a population of biological things (people, plants, etc.) or manufactured products, etc.

The Normal Distribution is a continuous distribution from - INFINITY to +INFINITY and symmetrical about the mean. The total area under the curve is equal to one. The point of inflection on the curve is equal to one standard deviation from the mean m.

A random variable X has a normal distribution and it is referred to as a normal random variable, if and only if its probability density is given by:

$$Y = f(x) = \frac{1}{\sigma\,(2\pi)^{1/2}}\ exp(-1/2)\left[\frac{(X-\mu)}{\sigma}\right]^2$$